Analysis of a Permanent Magnet Synchronous Machine for bicycle applications

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***Index Terms*—EIEN20, Lund University.**

I. Introduction

THe aim of this paper is to assess and measure various design options for an electrical machine, powering and powered by a standard push bicycle. We will be using both calculation and simulation through FEMM and Matlab to test our parameters, as well as information from EIEN20 Assignments 3, 4 and 5. Ideally the machine will have the potential to be implemented in the real world, meaning we have a number of restrictions on size, weight, torque etc. However, we do not plan to take the project further than the simulation stage.

A. Specification

Our overarching aims for the project at conception were as follows:

- Conceptualize a realistic machine with maximum efficiency for size.
- Machine to be small enough to fit within bicycle wheel.
- Machine to be light enough to be unobtrusive to rider.
- Realistic RPM for pedals, motor and wheel.

B. Assumptions

We made assumptions about the rider of the bike and the type of bike being ridden, all of our assumptions are based on general averages for each given quality.

- Rider weight=80Kg
- Bike weight=20Kg
- Wind speed=0mph

C. Restrictions

1) Wheel size: Our machine must be smaller than the wheel diameter. It may be slightly wider than the rim but this width will depend on our simulation results, it shouldn't protrude too far from the bike for practical reasons.

- Total wheel diameter=711mm (28")
- Rim depth=50mm
- Rim width=40mm
- Fork width=130mm

II. Calculation

A. Initial Study

Initially, it was needed to estimate the required power and torque that the machine needed to supply.
In order to do this, a Matlab script was created estimating the resisting air drag, roll resistance and gravitational resistance (when traveling in a positive or negative slope).
This was done using some arbitrary physical equations with rough parameters and coefficients.

Estimating Air drag, Roll resistance and Gravitational resistance:

$$F_{airdrag} = \frac{1}{2} * C_{drag} * A_{frontal} * \rho * v^2 \quad (1)$$

$$F_{roll} = C_{roll} * m * g * \cos\theta \quad (2)$$

$$F_{grav} = \sin\theta * m * g \quad (3)$$

The parameters and coefficients used were as follows:
Air drag coefficient

$$C_{drag} = 1.1 \quad (4)$$

Frontal area

$$A_{frontal} = 0.5m^2 \quad (5)$$

Density of the fluid (air)

$$\rho = 1.25\frac{kg}{m^3} \quad (6)$$

Roll coefficient

$$C_{roll} = 0.004 \quad (7)$$

Mass

$$m = 100kg \quad (8)$$

Graviational acceleration

$$g = 9.81\frac{m}{s^2} \quad (9)$$

The variables used were the speed of the bicycle, v, and the inclination of path travelled, θ.
The power and torque requirements were estimated using the formulas below, where v is the speed and ω the angular velocity of the wheel. Note that the requirements were overestimated by a 80 percent efficiency of transmission (outside of the machine).

$$P_{required} = \frac{(F_{airdrag} + F_{roll} + F_{grav}) * v}{0.8} \quad (10)$$

$$T_{required} = \frac{P_{required}}{\omega} \quad (11)$$

The first calculations we performed were to find the required torque and power from our motor in order to initialize further investigation. We used Matlab to write a program that takes some basic parameters and outputs torque and power. For example; to travel 10Km/h with an inclination of 3 degrees and efficiency 0.8 our program outputs a required power of 201.0W and a torque of 25.7Nm. For the case of no inclination, a speed of $\approx$ 27km/h can be achieved with 130W power and a torque of 6Nm.

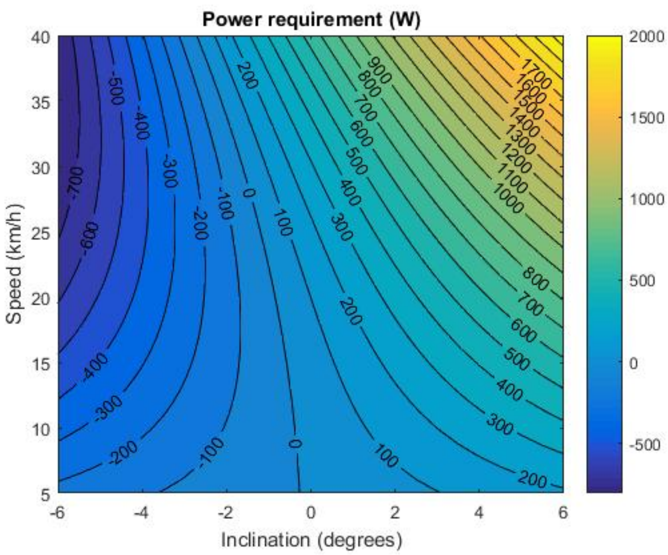

Fig. 1. Bicycle speed dependence on inclination angle and power

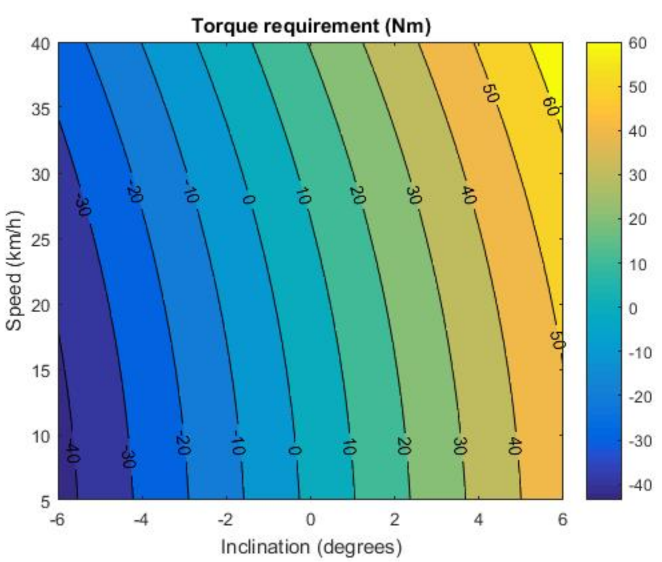

Fig. 2. Bicycle speed dependence on inclination angle and torque

B. Thermal Equivalent Circuit

The figures (3, 4, 5 and 6) depict a Matlab machine performance optimization from a thermal equivalent circuit for machine weight, torque, power and efficiency respectively.

- Stator thickness to machine diameter ratio = 0.2
- Inner rotor radius to machine radius ratio = 0.2
- No. of poles = 6
- Air gap width = 0.8mm

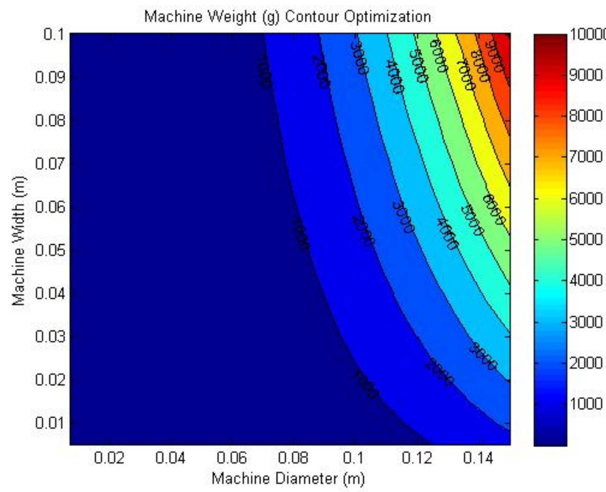

Fig. 3. Machine Weight Contour for width and diameter

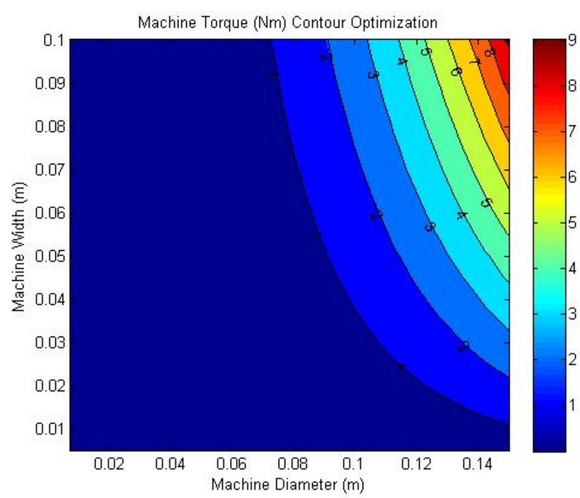

Fig. 4. Machine Torque Contour for width and diameter

1) Analysis: Because these simulations are equivalent circuit they are relatively inaccurate (compared to finite element) but they helped us to refine our choices for machine width and diameter. Our power goal is around 250W so we can use the simulation results to find a number of option areas for size that give our 250W output. Figure (3) shows that the weight of the machine is only a small consideration as, for these dimensions, the maximum is only around 4Kg which is significantly smaller than the bike and rider weight. Figures

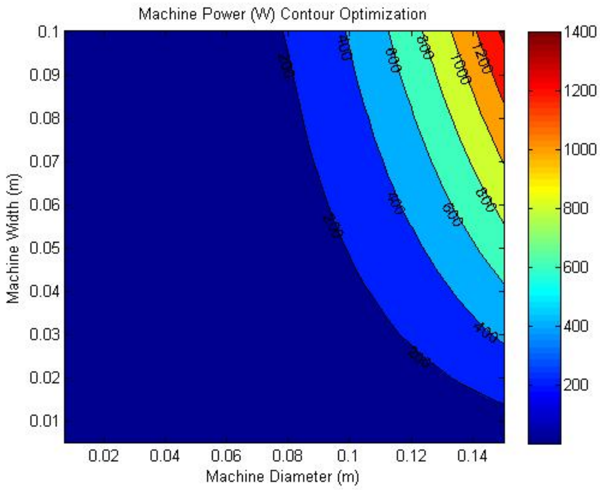

Fig. 5. Machine Power Contour for width and diameter

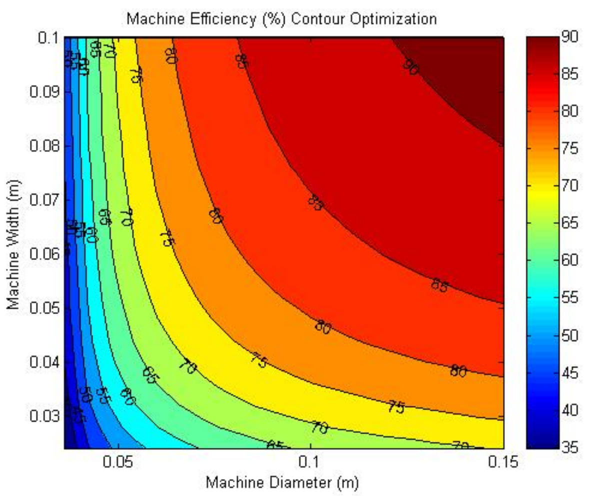

Fig. 6. Machine Efficiency Contour for width and diameter

(4) and (5) for torque and power are similar to each other, as we expected. For the power we require we can see that ideally our dimensions will fall somewhere on the dividing line between the dark and medium blues on the power graph, at around 250W. Figure 6 for efficiency shows us that slightly larger machines tend to be more efficient, we know this is not true at a larger scale but at this scale we should stay away from extremely small dimensions.

The study continued with the analysis of the effect of changing pole numbers and air-gap length on the machine performance in terms of torque, power and efficiency. The following starting values were used:

- Machine diameter = 90mm
- Machine width = 80mm
- Stator thickness to machine diameter ratio = 0.2
- Inner rotor radius to machine radius ratio = 0.2
- No. of poles = 6
- Air gap width = 0.8mm

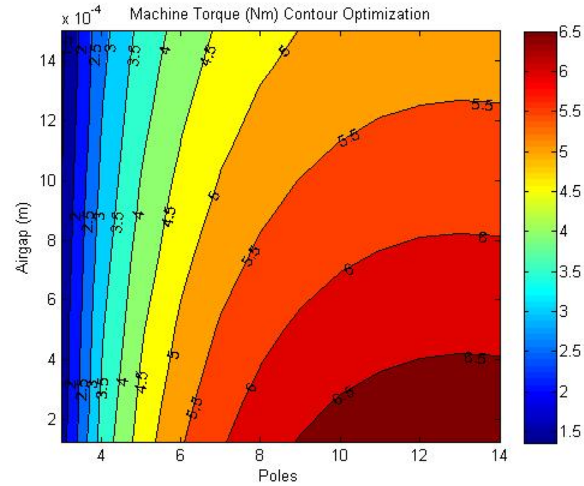

Fig. 7. Machine Torque Contour for Poles and Air Gap

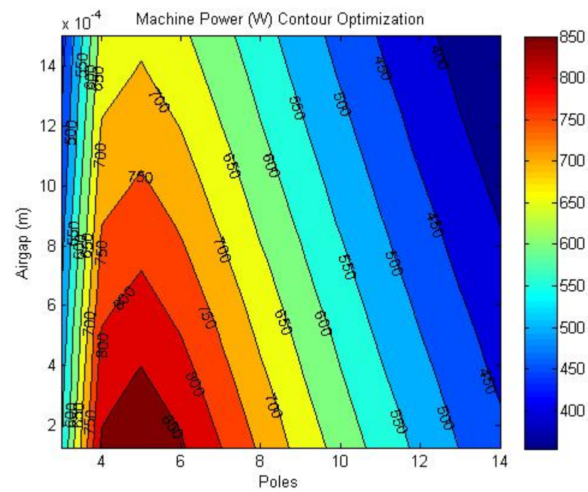

Fig. 8. Machine Power Contour for Poles and Air Gap

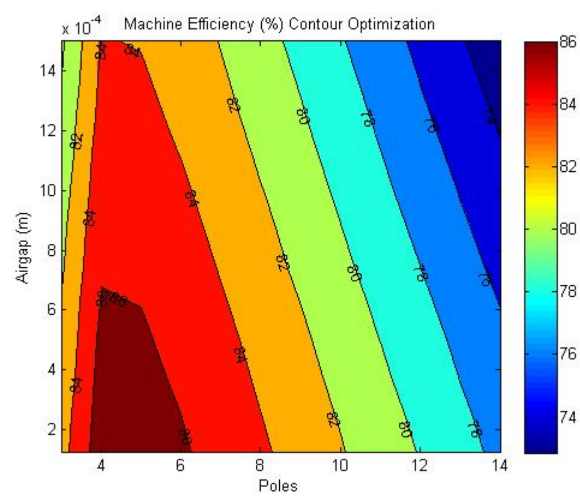

Fig. 9. Machine Efficiency Contour for Poles and Air Gap

2) Analysis: These figures (7, 8 and 9) give us a clear indication that the number of poles has a far stronger effect on the Torque, Power and Efficiency of the machine than the air gap width does. This will be a useful insight for final design decisions.

As both torque and speed can be adjusted using a planetary gear or varying the machine supply frequency, the machine power is the main design requirement we will use for determining the diameter and width. A Matlab script was written to determine the diameter and width for a required power from the contour plot of figure 5. With a required power of 200W, the script outputs optimal machine diameter of 75mm and width of 47.5mm. With these dimensions a 3 phase, 6 pole, 50Hz PMSM has a torque of 1.04Nm, total mass of 1.17kg and 3000 RPM. The resulting machine with planetary gear is shown in figures 10 and 11.

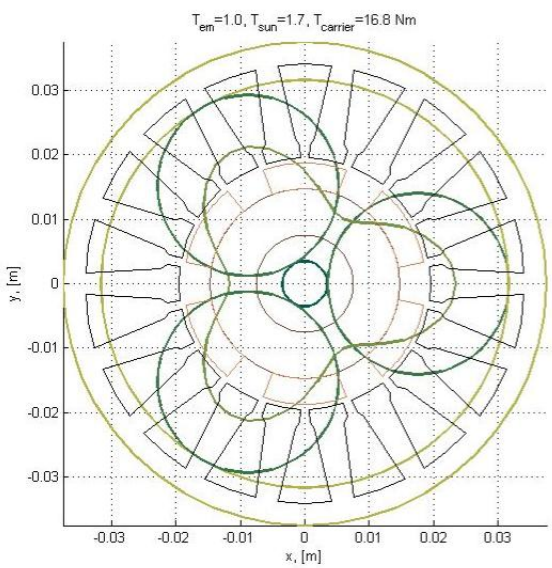

Fig. 10. Machine Dimension with Planetary Gear

C. FEMM Study

The study of optimal machine dimension was continued using the Finite Element Method (FEM) with a Matlab script calling the software FEMM. Both inner and outer rotor topologies were investigated separately. The stator thickness to diameter and inner rotor to diameter ratios were kept constant from the previous section.

Figures (12, 13, 14 and 15) represent the outer rotor machine torque, power, heat loss and efficiency respectively.

Figures (16, 17, 18 and 19) represent the inner rotor machine torque, power, heat loss and efficiency respectively.

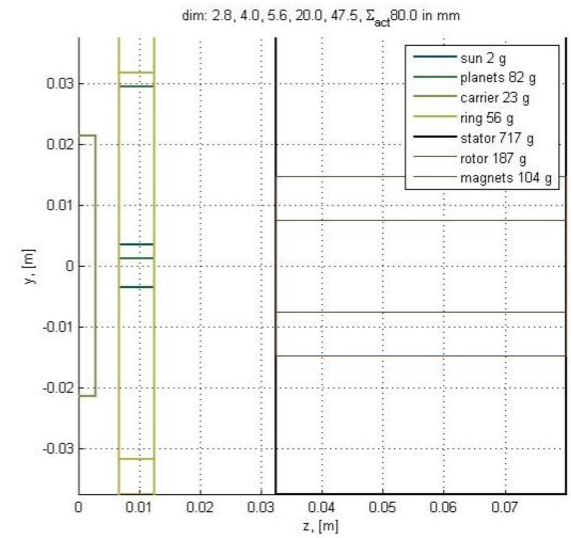

Fig. 11. Machine Cross Section with Planetary Gear

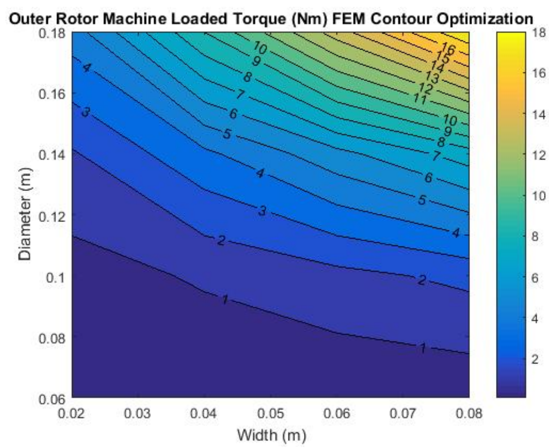

Fig. 12. Outer Rotor Machine Torque from FEM

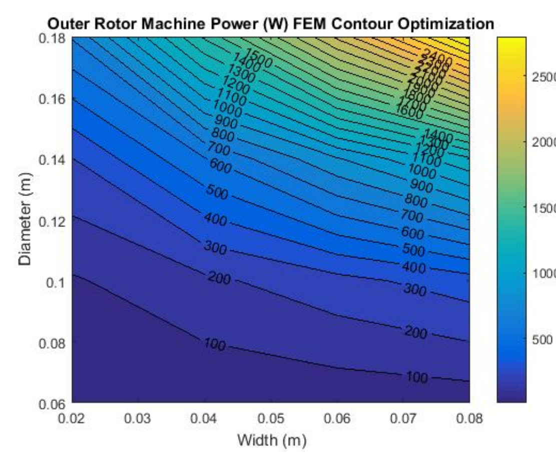

Fig. 13. Outer Rotor Machine Power from FEM

1) Analysis: A number of conclusions can be drawn from the FEM results, most markedly they show that there are

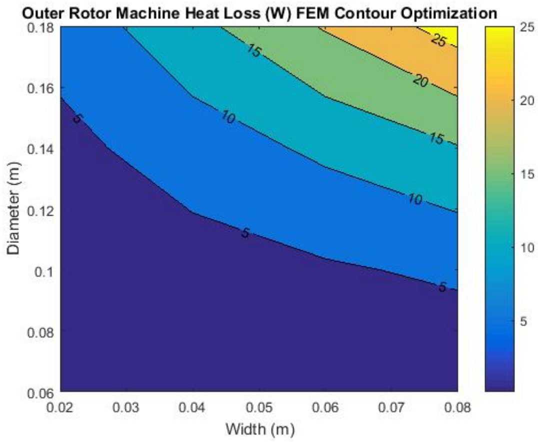

Fig. 14. Outer Rotor Machine Loss from FEM

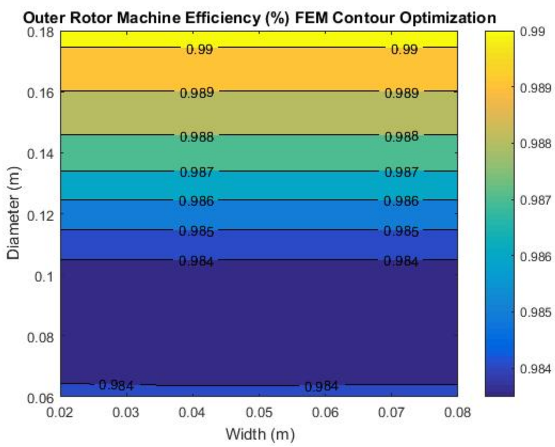

Fig. 15. Outer Rotor Machine Efficiency from FEM

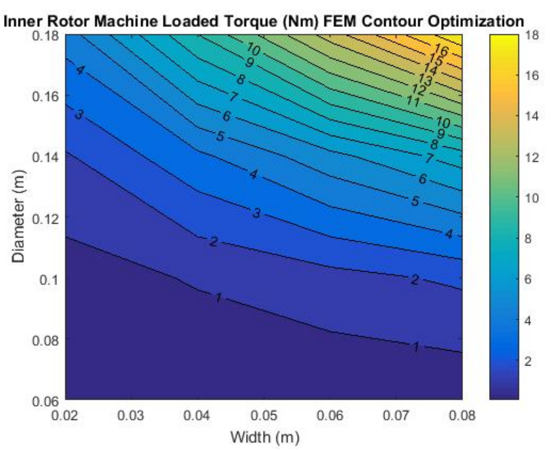

Fig. 16. Inner Rotor Machine Torque from FEM

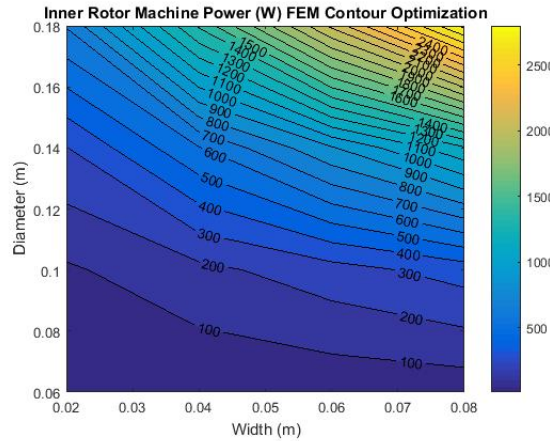

Fig. 17. Inner Rotor Machine Power from FEM

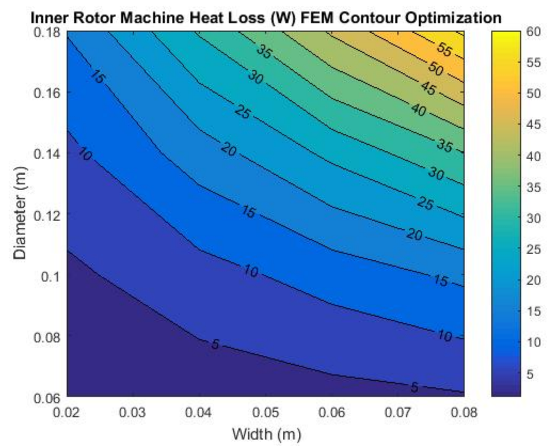

Fig. 18. Inner Rotor Machine Loss from FEM

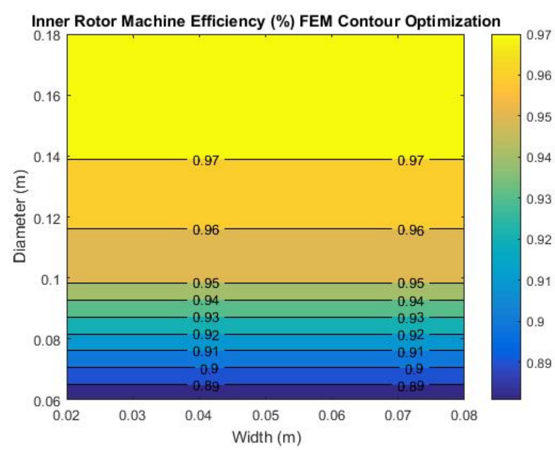

Fig. 19. Inner Rotor Machine Efficiency from FEM

significant differences between the FEM and equivalent circuit methods. One of these differences can be seen in the efficiency graphs for each, the equivalent circuit method shows efficiency as low as 35% whereas FEM shows the lowest efficiency as around 88% for inner rotor. We conclude that this difference is due to FEM being a purely 2-D simulation and therefor does not account for losses in the third dimension, which would become proportionally greater at low diameter values.

Another observation to be made is that there is very little difference between the results for Inner rotor topology and Outer rotor topology. The efficiency of the machine changes to a degree small enough that we can consider any of the specifications here to be efficient enough.

Therefor we can focus on Power and Torque, we will use the heat and power loss graphs in the next section to determine whether Inner or Outer rotor is best for our application.

D. Detailed FEM Inner and Outer Rotor Comparison

In this section, the inner and outer rotor topologies will be compared for the single geometry of 90mm of diameter and 80mm of width. The winding current density of both the inner and outer rotor topologies is 1.732 A/mm^2. Figures 20 and 21 depict the selected machine geometries. Figures 22 and 23 demonstrate the temperature distribution along the machine surface. Figures 24 and 25 show the flux density distribution along the machine surface. Figures 26 and 27 are for the flux density with angular position. Figures 28 and 29 show the flux density harmonics.

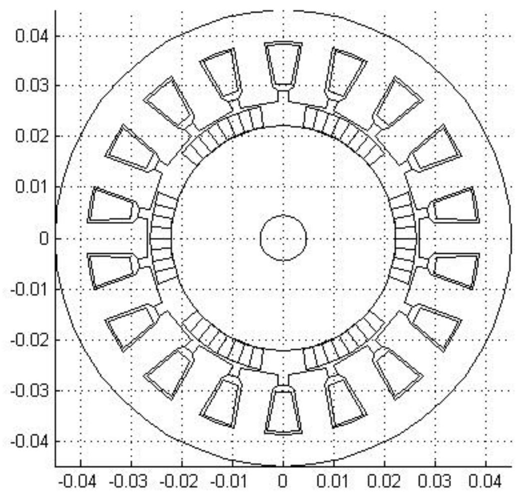

Fig. 20. Inner Machine Rotor Geometry

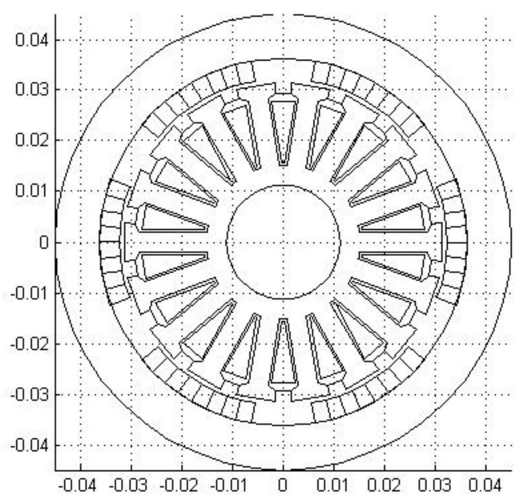

Fig. 21. Outer Machine Rotor Geometry

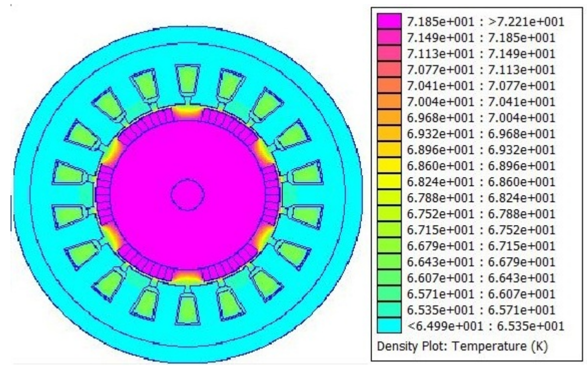

Fig. 22. Inner Machine Rotor Temperature Density

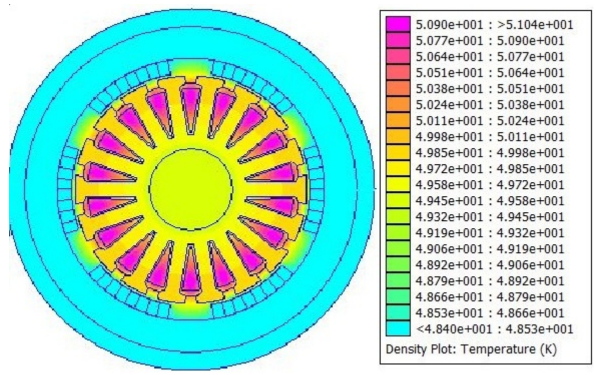

Fig. 23. Outer Machine Rotor Temperature Density

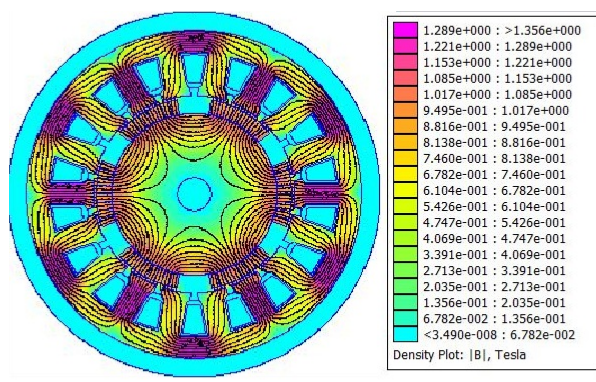

Fig. 24. Inner Machine Rotor Flux Density

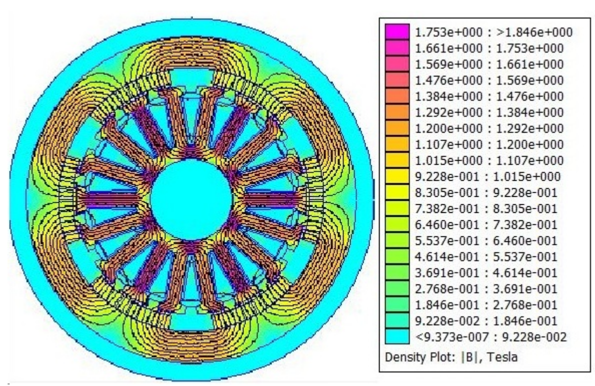

Fig. 25. Outer Machine Rotor Flux Density

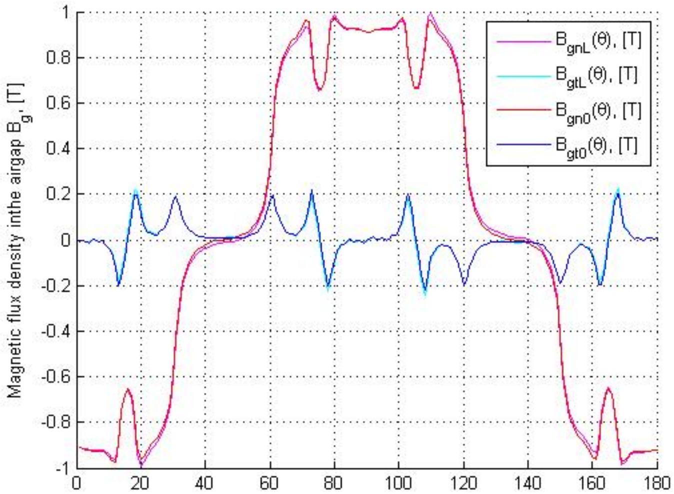

Fig. 26. Inner Machine Rotor Flux with Angular Position

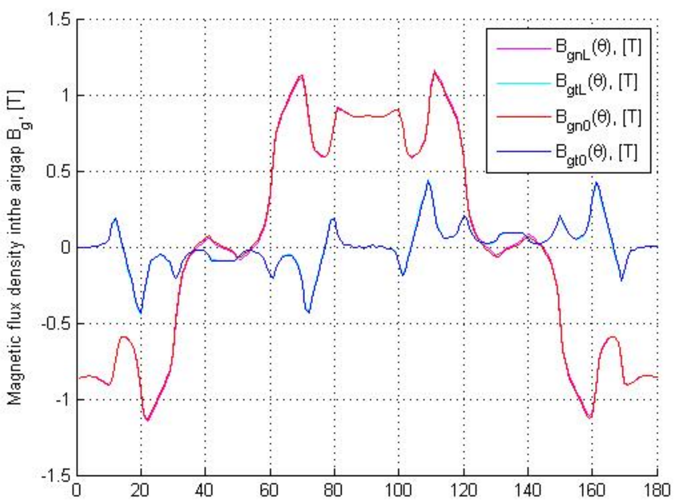

Fig. 27. Outer Machine Rotor Flux with Angular Position

1) Analysis: These figures allow us to compare and contrast the finer aspects of the inner and outer rotor constructions for our machine.

The temperature density images show us that the outer rotor machine has a lower maximum temperature than the inner rotor machine (50.9K and 71.85k). This is an advantage for the outer rotor design, more importantly the heat dissipating potential of the outer rotor design is greater than that of the inner rotor.

The flux density images show us that the outer rotor has a higher density of flux in the center compared to the relatively spread out flux density of the inner rotor. Both designs are large enough here that their maximum flux density remains below saturation. This is an important consideration for the size of our machine as a small machine will reach saturation in the core.

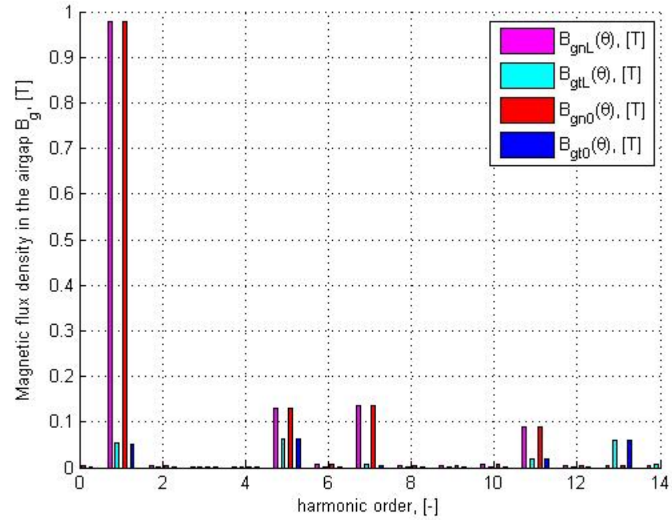

Fig. 28. Inner Machine Rotor Flux Harmonics

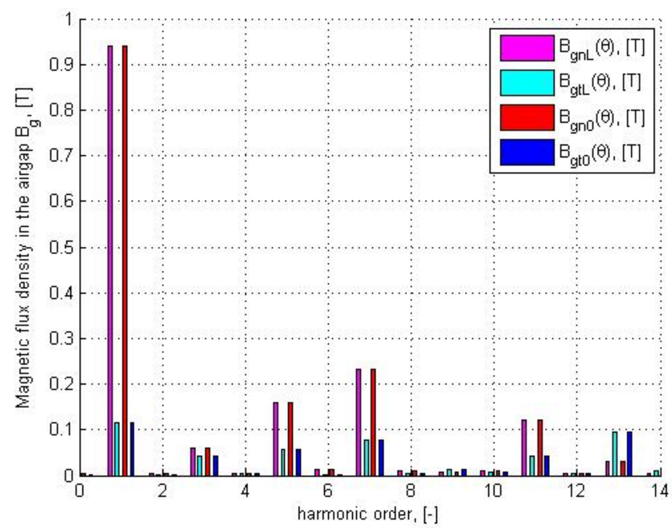

Fig. 29. Outer Machine Rotor Flux Harmonics

The differences in design choice here are not very great, the outer rotor has a better structure for heat dissipation and the inner for flux density distribution. One extra consideration is that our application requires the machine to be mounted on or around the rear hub of a bicycle. This would be best achieved with the outer rotor design as it matches the existing structure of the bike wheel.

III. Final Machine

Tables I, II and III illustrate the final machine quantities. Figures 31, 32 and 33, 34 are for the final machine heat and flux density plots at 1.7 and 4.3 A/mm^2 respectively, demonstrating other uses for our design.

TABLE I
Final Machine Parameters

Width	Diameter	Poles	Air Gap	Rotor Position	Weight
80mm	120mm	6	0.8mm	Outer	3.5Kg

TABLE II
Final Machine Output

Torque (Nm)	Current Density (A/m^2)
4.4	1.732
10.8	4.330

TABLE III
Final Machine Output

$\Psi_m(Vs)$	$L_{sx}(\mu H)$	$L_{sy}(\mu H)$	Saliency
0.0066	0.896	2.090	2.333

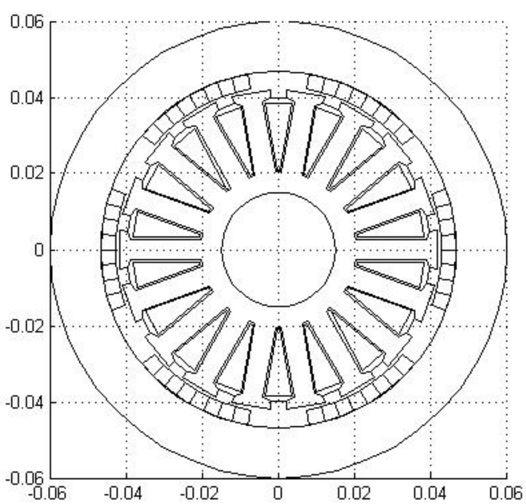

Fig. 30. Final Machine Geometry

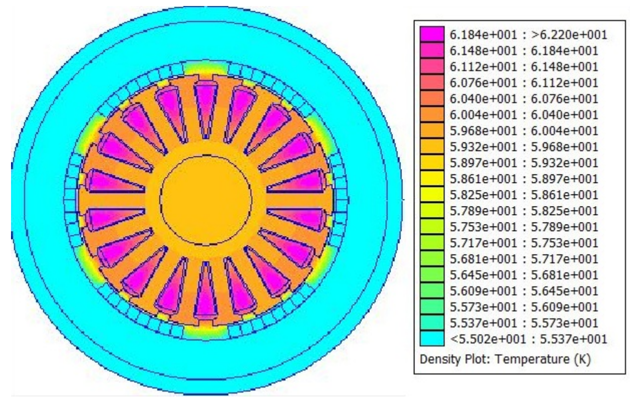

Fig. 31. Final Machine Heat Density for $1.7A/mm^2$

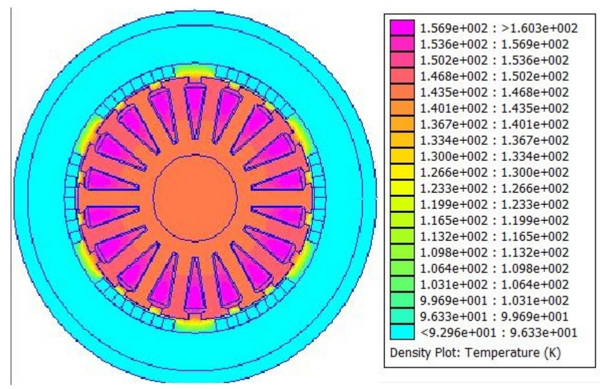

Fig. 32. Final Machine Heat Density for $4.3A/mm^2$

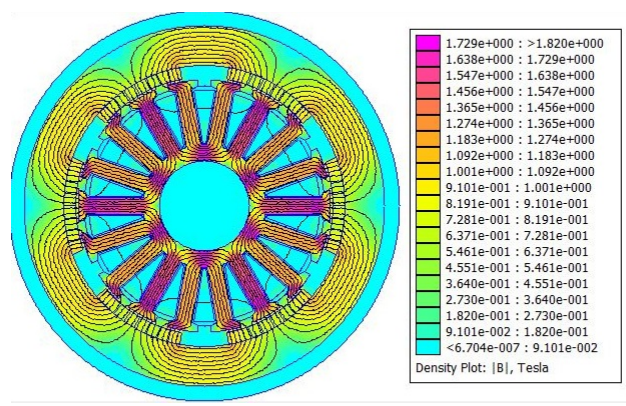

Fig. 33. Final Machine Flux Density for $1.7A/mm^2$

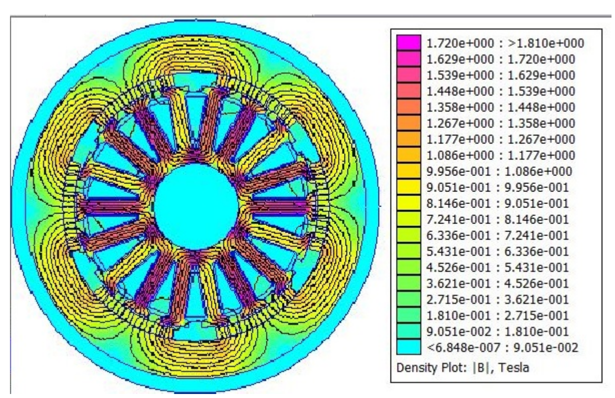

Fig. 34. Final Machine Flux Density for $4.3A/mm^2$

Figures 35 and 36 show the circle diagram and the torque/speed characteristics of the machine. The torque/speed graph indicates that this machine can not use field weakening to achieve higher speeds. This means our design would rely on regular or planetary gearing for speed and torque control, gearing systems would also help to reduce complexity of the electronic components of the machine.

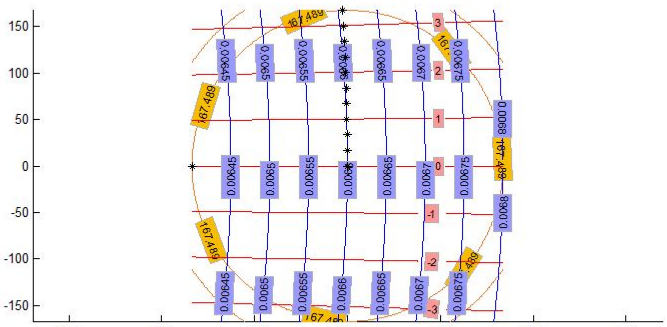

Fig. 35. Circle Diagram

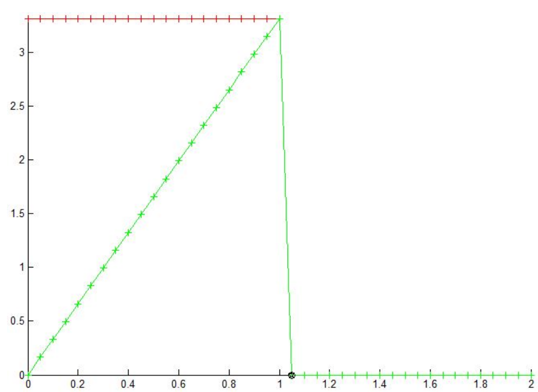

Fig. 36. Torque with Speed

IV. Conclusion

Our aim with this project was to identify a set of parameters that would be suitable for a Permanent Magnet Synchronous Machine mounted within a bicycle wheel. The machine should both provide torque as a motor and store power as a generator. Generally, we achieved this goal. The results above show our findings, we chose a relatively uniform, small machine to fit in the bike wheel with 6 poles and an outer rotor for efficiency and cooling purposes. Our machine will output a fairly steady 4.4Nm of torque which provides either pedal assistance or low speed full bike powering.

A. Continuation of study

If we were to continue the analysis of this machine further there are various areas of study that would prove useful in full optimization.

In terms of the machine model we would look at slot geometry, cooling optimization and current burst analysis. Material types and stress testing would provide a more advanced overall analysis model.

The power storage and battery system would be an area we could improve. This would involve researching the ideal battery size and structure for our application A seperate area of continued study could be alternate implementations for our machine. Some other low power applications could be investigated as potential projects.

Acknowledgment

Programs and websites used in writing this report:

- Matlab
- FEMM
- Bike Calculator

References

[1] A. Reinap, "Performance estimation for a three-phase PM synchronous machine, the 3rd home assignment in the course on design of electrical machines EIEN20, 2016.

[2] A. Reinap, "Finite element analysis of a three-phase PM synchronous machine, the 4th home assignment in the course on design of electrical machines EIEN20, 2016

[3] A. Reinap, "Machine characteristics, the 5th home assignment in the course on design of electrical machines EIEN20, 2016.